Year 12 Mathematics Methods Test 6
Sampling & Confidence Intervals

Name: SOLUTIONS

Section 1: Calculator Free 22 marks 20 minutes (maximum)

QUESTION 1 [7 marks]

A sample of students in Year 8 and Year 11 at CVC was sent a survey via Connect to help decide school priorities. There were 320 students in Year 8 and 280 in Year 11 at the school. The students were selected on a **systematic** basis using the Library card numbers. The Year 8 students start at 365 and the Year 11s start at 1841. The ID numbers are consecutive and none are missing. The first Year 8 student selected had an ID number of 677 and the first Year 11 student had an ID number of 1957. In total, 120 students were selected.

a) Describe two likely problems with the survey

Some may choose not to respond ✓
Only yr8s and yr11s surveyed ✓

b) How many students in Year 8 should be selected?

$\frac{320}{600} \times 120$ ✓

$= 64$ ✓

c) What were the ID numbers of the next four Year 8 students selected?

$\frac{320}{64} = 5 \Rightarrow$ go up in 5s ✓ Last yr 8 has Number 684 ✓

682, 687, 372, 377 ✓

QUESTION 2 [4 marks]

A sample of n items is found to have m items satisfying a particular property. Show that the standard deviation of the sampling distribution is given by $\frac{1}{n}\sqrt{m - \frac{m^2}{n}}$

$\hat{p} = \frac{m}{n}$ ✓

$\therefore \sigma = \sqrt{\frac{(\frac{m}{n})(1-\frac{m}{n})}{n}}$ ✓

$= \sqrt{\frac{\frac{m}{n} - \frac{m^2}{n^2}}{n}}$

$= \sqrt{\frac{m}{n^2} - \frac{m^2}{n^3}}$ ✓

$= \sqrt{\frac{1}{n^2}\left(m - \frac{m^2}{n}\right)}$ ✓ $= \frac{1}{n}\sqrt{m - \frac{m^2}{n}}$

QUESTION 3 [1, 2, 3 marks]

Of the 600 people on a train carriage on the Mandurah line, 240 were travelling past Cockburn.

a) What is the sample proportion of passengers travelling past Cockburn? (as a decimal)

$\frac{24}{60} = 0.4$ ✓

b) What is the standard deviation? (as a decimal)

$\sqrt{\frac{0.4 \times 0.6}{600}}$ ✓ $= 0.02$ ✓

c) What is the 95% confidence interval (z = 1.96) for the probability of a passenger travelling past Cockburn?

$0.4 - 1.96 \times 0.02 \leq p \leq 0.4 + 1.96 \times 0.02$ ✓

$0.4 - 0.0392 \leq p \leq 0.4 + 0.0392$

$0.3608 \leq p \leq 0.4392$ ✓ ✓

$0.0196 \times 2 = 0.0392$

QUESTION 4 [3 marks]

The ages of 80 people towing caravans in the north of Western Australia were checked in June and 24 were found to be over the age of 60.

a) Estimate the probability of caravan drivers being over the age of 60. (as a decimal)

$\frac{24}{80} = 0.3$ ✓

b) State any problems with this estimate

Biased ✓, likely to be 'grey nomads' ✓ (or other bias)

QUESTION 5 [2 mark]

Describe what effect decreasing the sample size will have on the length of the confidence interval, if you hold the confidence level the same?

Increase the length. ✓

When dividing by n, when n decreases $\sqrt{\frac{p(1-p)}{n}}$ must increase. ✓

CANNING VALE COLLEGE

Year 12 Mathematics Methods Test 6
Sampling & Confidence Intervals

Name: Solutions

Section 2: Calculator Assumed 38 marks 35 minutes (maximum)

QUESTION 6 [1, 2 marks]

35 out of 50 people working in the city were wearing dark-coloured clothing.

a) Estimate the 95% confidence interval for the probability of city workers wearing dark clothing.

$0.573 \leq p \leq 0.827$ ✓

b) Interpret your answer

95% of such samples will contain ✓✓ the population proportion

QUESTION 7 [2 marks]

It is generally assumed that the chances of a baby being born being a girl or boy are 50-50. How many births would need to be checked to establish the true proportion in Australia to within 0.1% at a confidence level of 95%?

$z = 1.96$ $p = 0.5$ $e = 0.001$ ✓

$n = 960\,400$

Need to check 960 400 births ✓

QUESTION 8 [1, 1, 1 marks]

A newspaper determined that an approximate 95% confidence interval for the proportion of people in Australia who regularly read the news online was between 0.50 and 0.70

a) What proportion of people in their survey read the news online?

0.6 ✓

b) What is the margin of error?

0.1 ✓

c) How could the newspaper increase the precision of their study?

Survey more people ✓

QUESTION 9 [2, 1, 1, 3 marks]

A survey of 300 voters selected at random from Busselton claimed that the WA vote for a new political party would be 15% ± 3%.

a) Comment on the method used in this survey

Biased – only 1 electorate ✓
Small sample ✓

b) What is the margin of error in the result?

0.03 ✓

c) What is the sample proportion given in the result?

0.15 ✓

d) How should a survey be conducted to establish the likely WA vote for the new party to an accuracy of 3% at a level of confidence of 95%?

$1.96\sqrt{\frac{0.15 \times 0.85}{n}} = 0.03$ ✓

$n = 544.2$ ✓

∴ Survey 545 people from all over WA ✓

QUESTION 10 [1, 2, 2 marks]

A sample of *n* people was asked whether they thought that income tax in Australia was too high, and 90% said yes.

a) What is the value of the sample proportion, $\hat{p}$?

0.9 ✓

b) Write a simplified expression for M, the margin of error for this estimate at the 95% confidence interval, in terms of *n*

$1.96\sqrt{\frac{0.9 \times 0.1}{n}} = \frac{1.96 \times 0.3}{\sqrt{n}} = \frac{0.588}{\sqrt{n}}$ ✓ ✓

c) If the number of people in the sample were doubled, what would be the effect on the margin of error, M?

Reduced ✓ by a scale factor of $\sqrt{2}$ ✓

QUESTION 11 [1, 1, 2, 2 marks]

A landscape gardener wants to know how many carp are in a large ornamental lake. He decides to use capture-recapture sampling.

a) Suppose there are N carp in the lake and he captures 500 of them, tags them then releases them back into the lake. Write an expression for the proportion of tagged carp in the lake.

$\frac{500}{N}$ ✓

b) The next day, a sample of 400 carp is captured from the lake, and he finds that there are 60 tagged carp in the sample. What is the proportion of tagged carp in the second sample?

$\frac{60}{400}$ ✓ (0.15)

c) Use the fact that this proportion should be approximately the same as the proportion in the lake to estimate the number of carp in the lake

$\frac{500}{N} = \frac{60}{400}$ ✓

$N = 3333.3$ ✓

∴ Approx 3333 carp

d) Find the approximate 99% confidence interval for the number of carp in the lake.

$0.104 < p < 0.196$ ✓

$\therefore \frac{500}{0.196} < N < \frac{500}{0.104}$

$2551 < N < 4808$ (or 4807) ✓

QUESTION 12 [4, 3 marks]

Tarun is shooting arrows at a target which he has a probability of 0.6 of hitting. What is the approximate probability that:

a) The proportion of time that he hits the target in his next 100 attempts is less than 0.8?

$np > 10$
$n(1-p) > 10$ ✓

$\therefore X \sim N(0.6, \sqrt{0.0024})$ ✓

$P(X < 0.8) = 1$ ✓

b) The proportion of time that he hits the target in his next 100 attempts is between 0.7 and 0.8 given that it is more than 0.6?

Look, Mo

$P(0.7 < X < 0.8 \mid X > 0.6) = \frac{0.0206}{0.5}$ ✓ ✓

$= 0.0412$

QUESTION 13 [2, 3 marks]

Seeds are sold in packets of 50. The wholesaler guarantees that 99% of the seed will germinate. Cameron selects and tests ten samples of 50 packets at random. The number of seeds that don't germinate in each of the 10 random samples is shown below:

Sample	1	2	3	4	5	6	7	8	9	10
Number of seeds not germinated	34	27	22	29	28	36	22	29	28	20

a) Using the assumption that 99% of the seeds will germinate, calculate the 95% confidence interval for the proportion of non-germinating seeds expected when sampling

Samples : $50 \times 50 \times 10$
$= 25000$ ✓

Not germ = 250

$0.00877 < p < 0.01123$ ✓

b) Explain which, if any, of the samples lie outside the 95% confidence interval

$0.01123 \times 2500 = 28.075$

$0.00877 \times 2500 = 21.92$

Samples < 21.9 ✓ or > 28.1 ✓
are out of confidence interval

ie Samples 1, 4, 6, 8, 10 ✓